PROBLEM 1: LINK-BUDGET (ELASTIC-BACKSCATTER LIDAR)

Consider an elastic-backscatter lidar defined by the following system parameters:

LASER
- Quantel (Nd:YAG 2ω)
- Emission wavelength: 532 nm
- Energy, E: 160 mJ
- Pulse-repetition frequency, PRF: 20 Hz
- Beam width, \( w_0 \): 5\( \times\)10\(^{-3} \) m (*)
- Divergence (half-width angle), \( \theta_{1/2} \): 0.5 mrad (*)

TELESCOPE
- Celestron Schmidt-Cassegrain C-8, 8" (0.2032 m)
- Shade diameter, \( d_{sh} \): 2.7" (0.06858 m)
- Focal length, f: 2 m
- Transmissivity, \( T_1 \): 60 %

OPTO-ELECTRONIC RECEIVER

PHOTODIODE
- APD (EGG C30956E)
- Active area diameter, \( D_{APD} \): 3 mm
- Multiplication factor, M: 150
- Excess-noise factor, F: 4.5
- Dark surface current, \( I_{ds} \): 7.64\( \times\)10\(^{-8} \) A
- Dark bulk current, \( I_{db} \): 3.10\( \times\)10\(^{-10} \) A
- Intrinsic responsivity, \( R_{io} \): 240 mA/W

INTERFERENCE FILTER
- Bandwidth, \( \Delta \lambda \): 10 nm
- Transmissivity, \( T_2 \): 65 %

SIGNAL-CONDITIONING STAGES
- Transimpedance Gain (1\(^{st} \) stage), \( G_t \): 5750 \( \Omega \)
- Voltage conditioning Gain (2\(^{nd} \) stage), \( G_{ac} \): 20.3 V/V
- Noise-equivalent bandwidth, B: 10 MHz
- Equivalent input noise (chain input), \( \sigma_{th,i} \): 5 pA\( \cdot\)Hz\(^{-1/2} \)

ATMOSPHERE

Aerosol component:
- Visibility margin (532 nm), \( V_M \): 39.12 km
- Lidar ratio, \( S=\alpha_{Mie}/\beta_{Mie} \): 25 sr
- Boundary-layer height, \( R_{PBL} \): 3 km

Molecular component (average):
- Rayleigh’s extinction: 0.01 km\(^{-1} \)
- Rayleigh’s ratio \((\alpha_{Ray}/\beta_{Ray})\): 8\( \pi/3 \)
Background-radiance component
- Moon’s radiance (full Moon), \( L_{\text{Moon}} \): \( 3 \times 10^{-11} \, \text{W} \cdot \text{cm}^{-2} \cdot \text{nm}^{-1} \cdot \text{sr}^{-1} \)
- Solar radiance, \( L_{\text{Sun}} \) (typ.): \( 3 \times 10^{-6} \, \text{W} \cdot \text{cm}^{-2} \cdot \text{nm}^{-1} \cdot \text{sr}^{-1} \)

OTHER PARAMETERS
- Full-overlap range, \( R_{\text{ovf}} \): 200 m
- Maximum-range criterion \( \text{SNR}(R_{\text{max}}) = 1 \)

PHYSICAL CONSTANTS
- Electron charge, \( q \): \( 1.602 \times 10^{-19} \, \text{C} \)
- Planck’s constant, \( h \): \( 6.6262 \times 10^{-34} \, \text{J} \cdot \text{s} \)
- Light speed, \( c \): \( 2.99793 \times 10^8 \, \text{m} \cdot \text{s}^{-1} \)
- Boltzmann’s constant, \( K \): \( 1.38 \times 10^{-23} \, \text{J} \cdot \text{K}^{-1} \)

(*) Parameter not used.

Questions:
1. Determine the system constant, \( K(\lambda) \) [W·km\(^3\)]
2. Estimate the received background power under night-time operation, \( P_{\text{back}} \)
3. Compute and plot the return power at the following ranges:
   a. \( P(0.2 \, \text{km}) \)
   b. \( P(1 \, \text{km}) \)
   c. \( P(2 \, \text{km}) \)
   d. \( P(3 \, \text{km}) \)
   e. \( P(3^* \, \text{km}) \)
   f. \( P(4 \, \text{km}) \)
4. Compute the receiver-chain voltage responsivity, \( R_v \), and the net voltage responsivity (i.e., including spectral optical losses), \( R_v' \).
5. a) Compute the range-dependent signal-to-noise ratio (consider the ranges of question 3), \( \text{SNR}(R) \).
    b) Identify the noise-dominant system-operation mode.
6. Assess the approximate laser-radar maximum range.
7. How many pulses are needed to integrate in order to ensure a \( \text{SNR}_v \) (voltage signal-to-noise ratio) of 40 dB at 3-km range? What is the resulting observation time of the lidar instrument?
8. Now, consider a Raman system of similar specs. If for Raman systems the return signal is typically 3 orders of magnitude lower than for their elastic system counterparts, discuss on the feasibility of day-time operation.
9. Compute the photodiode NEP and its quantum efficiency (\( \eta \)).
10. Compute the system NEP (\( \text{NEP}_s \)).
SOLUTIONS

1. \( K(532 \text{ nm}) = 6.89 \times 10^{-4} \text{ W}\cdot\text{km}^3 \)

2. \( P_{\text{back}} = 1.52 \times 10^{-13} \text{ W} \)

3. \( P(0.2 \text{ km}) = 8.56 \times 10^{-5} \text{ W}, \ P(1 \text{ km}) = 2.87 \times 10^{-6} \text{ W}, \ P(2 \text{ km}) = 5.76 \times 10^{-7} \text{ W}, \ P(3 \text{ km}) = 2.06 \times 10^{-7} \text{ W}, \ P(3^* \text{ km}) = 4.72 \times 10^{-8} \text{ W}, \ P(4 \text{ km}) = 2.60 \times 10^{-8} \text{ W} \)

4. \( R_v = 4.20 \times 10^6 \text{ V/W}, \ R_v' = 1.64 \times 10^6 \text{ V/W} \)

5. \( SNR_v(0.2 \text{ km}) = 57.5 \text{ dB}, \ SNR_v(1 \text{ km}) = 42.7 \text{ dB}, \ SNR_v(2 \text{ km}) = 35.7 \text{ dB}, \ SNR_v(3 \text{ km}) = 31.3 \text{ dB}, \ SNR_v(3^* \text{ km}) = 24.9 \text{ dB}, \ SNR_v(4 \text{ km}) = 22.3 \text{ dB} \)

6. \( R_{\text{max}} = 16.9 \text{ km} \)

7. \( \eta = 8 \text{ pulses}, \ t_{\text{obs}} = 0.4 \text{ s} \)

8. The Raman lidar cannot be operated day-time because \( P(0.2 \text{ km}) = 8.56 \times 10^{-8} \text{ W} \) is progressively comparable to the background component, \( P_{\text{back}} = 1.52 \times 10^{-8} \text{ W} \), as we move further in range.

9. \( \text{NEP} = 88.2 \text{ fW}\cdot\text{Hz}^{-1/2}, \ \eta = 55.9 \% \)

10. \( \text{NEP}_s = 422 \text{ fW}\cdot\text{Hz}^{-1/2} \)