Except from an exam
Consider the vector space $\mathbb{R}^2$ with its standard basis $B$, and we consider the basis $B' = (u_1, u_2)$
$u_1 = (1, 2)$ \_B and $u_2 = (1, 1)$ \_B.
Consider the vector $v = (-2, -1)$ \_B.
Plot basis $B$
Plot basis $B'$
Plot $v$ \_B
Plot $v$ \_B'
What do you observe?

Answer: $v_{B'} = D^{-1} v_{B}$

$D = (u_1, u_2) = (1, 1)$
Thus $D^{-1} = (-1, 1)$

$D^{-1} = (2, -1)$
$V_{B_1}^B = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$V_{B_1}^B = (1, -3)$

$V_{B_1}^B = u_1 - 3u_2 \quad \mathbb{R}^2$

$B = (e_1, e_2)$ with

$e_1 = (1, 0) _B$

$e_2 = (0, 1) _B$

$V_{B_1}^B$ and $V_{B_1}^B$ are the same vector.
Student's answer

\[ V_{1B'} = (1, -3) |_{B'} \]