PERFORMANCE ANALYSIS OF SYNCHRONIZERS

TRACKING PERFORMANCE

FEEDBACK SYNCHRONIZERS

Error detector characteristic $S_{\varphi}(\phi) \triangleq E\left\{e_{\varphi}(k;\hat{\varphi})\right\} \quad \phi \triangleq \varphi_0 - \hat{\varphi}$

Error detector output

Equilibrium point: $\phi = 0 \Rightarrow S_{\varphi}(0) = 0$

Zero mean loop noise: $N_{\varphi}(k;\hat{\varphi}) = e_{\varphi}(k;\hat{\varphi}) - S_{\varphi}(\varphi_0 - \hat{\varphi})$

$$R_{\varphi}(k;\phi) = E\left[N_{\varphi}(m;\hat{\varphi})\,N_{\varphi}(m+k;\hat{\varphi})\right]$$

$$S_{\varphi}\left(\exp(jwT);\phi\right) = \sum_{k=-\infty}^{\infty} R_{\varphi}(k;\phi)\exp(-jwkT)$$

Linearized tracking performance:

$$\phi(k) = -\frac{1}{K_\varphi}\sum_m h_\varphi(k-m)N_\varphi(m;\varphi_0)$$

$$\mathrm{var}[\phi] = \frac{1}{K_\varphi^2} T \int_{-\pi/T}^{\pi/T} \left|H_\varphi(\exp(jwT))\right|^2 S_\varphi(\exp(jwT);0)\frac{dw}{2\pi}$$

$$K_\varphi = \left.\frac{\partial S_\varphi(\phi)}{\partial \phi}\right|_{\phi=0}$$

Slope of the phase error detector characteristic at the stable equilibrium point

$H_\varphi(\exp(jwT))$ Close-loop tranfer function of the synchronizer

FEEDFORWARD SYNCHRONIZERS

$$L(\hat{\varphi}) = \max_{\varphi} L(\varphi)$$

$$L(\varphi) ; L(\varphi_0) + (\varphi - \varphi_0) L'(\varphi_0) + \frac{1}{2}(\varphi - \varphi_0)^2 L''(\varphi_0)$$

$$\hat{\varphi} - \varphi_0 = -\frac{L'(\varphi_0)}{L''(\varphi_0)}$$

The statistical fluctuation of $L''(\varphi_0)$ with respect to its mean value is much smaller than this mean value:

$$\hat{\varphi} - \varphi_0 = -\frac{L'(\varphi_0)}{E\left[L''(\varphi_0)\right]}$$

$$\mathrm{var}[\phi] = \frac{E\left[\left(L'(\varphi_0)\right)^2\right]}{\left(E\left[L''(\varphi_0)\right]\right)^2}$$

$$L(\varphi)=\sum_{k=0}^{L_0-1} L(k;\varphi)$$

FEEDFORWARD: $$\phi=\frac{1}{E\left[L''(k;\varphi_0)\right]}\frac{1}{K}\sum_{k=0}^{L_0-1} L'(k;\varphi_0)$$

FEEDBACK: $$e_\varphi(k;\hat{\varphi})=L'(k;\hat{\varphi})$$

$$\phi=0 \quad \begin{cases} K_\varphi = E\left[L''(k;\varphi_0)\right] \\ N_\varphi(k;\varphi_0)=L'(k;\varphi_0) \end{cases}$$

$$E\left[L'(k;\varphi_0)\right]=0$$

$$\phi(k)=-\frac{1}{E\left[L''(k;\varphi_0)\right]}\sum_m h_\varphi(k-m)L'(m;\varphi_0) \longrightarrow h_\varphi(m)=\begin{cases} 1/K & m=0,1,\ldots K-1 \\ 0 & otherwise \end{cases}$$

COMPARISON OF SYMBOL SYNCHRONIZERS

The tracking performance depends on the shape of the received baseband pulse.

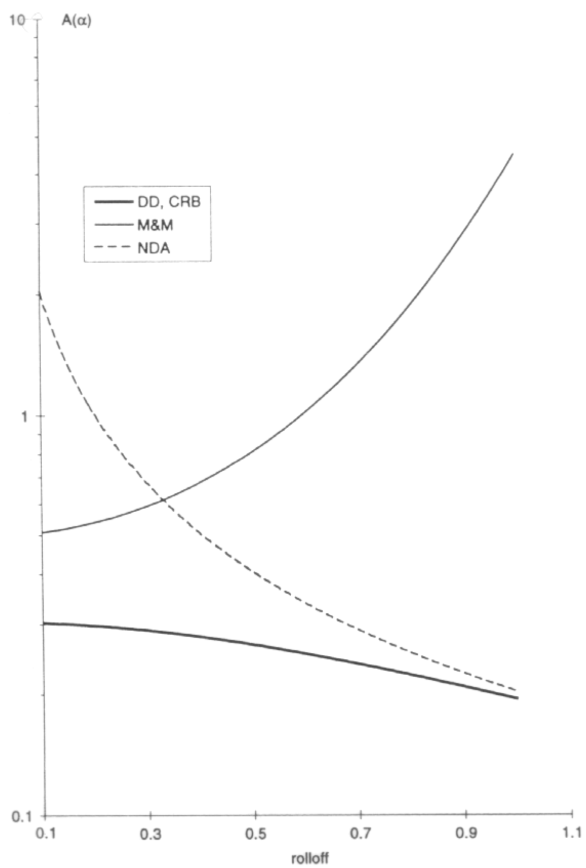
In obtaining numerical results it will be assumed that the output of the matched filter is a raised cosine pulse.

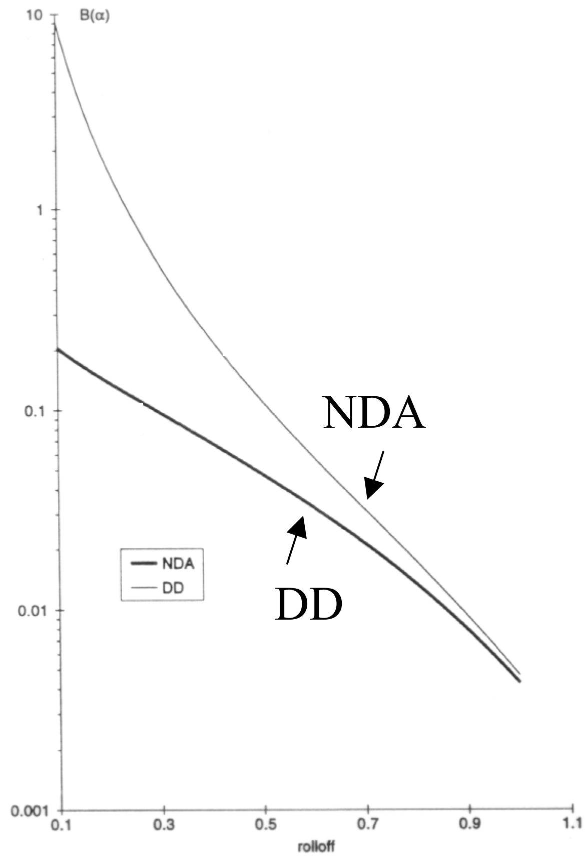
For moderated and large $\frac{E_s}{N_0}$, the timing error variance is well approximated by:

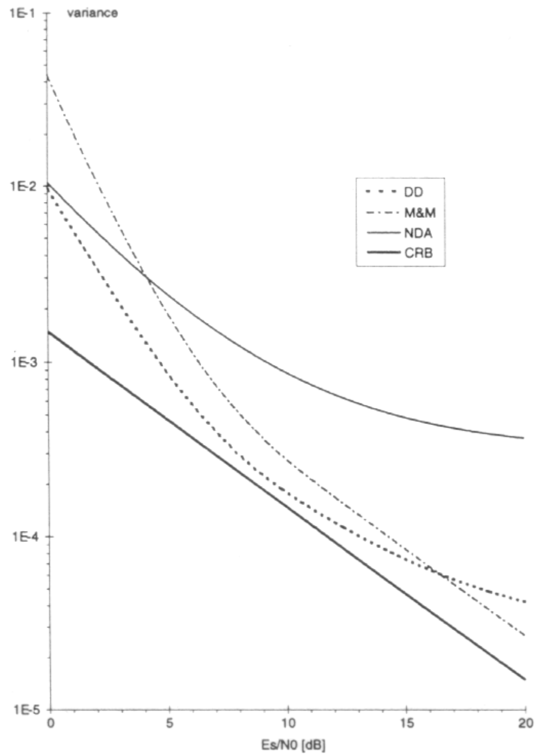
$$\operatorname{var}[e] \simeq \underbrace{(2B_LT)A(\alpha)\frac{N_0}{2E_s}}_{\text{Contribution from additive noise}} + \underbrace{K_F(2B_LT)^2 B(\alpha)}_{\text{Contribution from self noise}}$$

Contribution from additive noise   Contribution from self noise

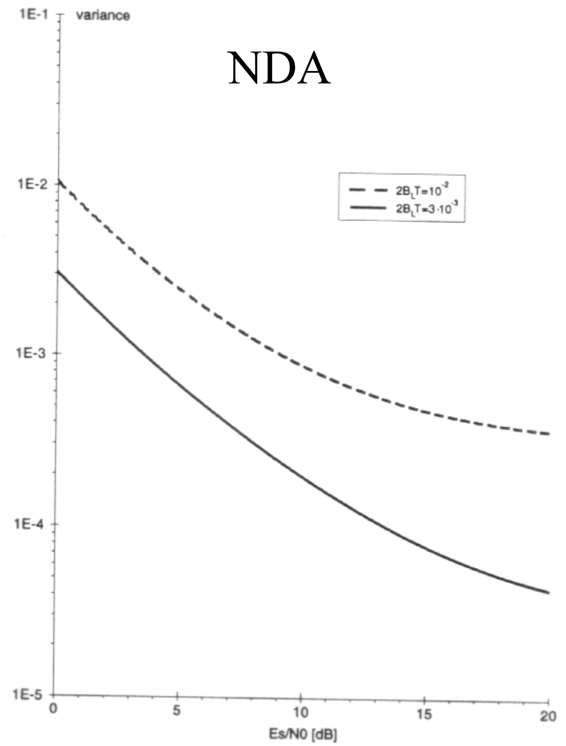
Rolloff factor: α


A(α)
DD, CRB
M&M
NDA
rolloff


B(α)
NDA
DD
NDA
DD
rolloff


1E-1
variance
1E-2
1E-3
1E-4
1E-5
DD
M&M
NDA
CRB
0
5
10
15
20
Es/N0 [dB]

NDA


1E-1
variance
1E-2
1E-3
1E-4
1E-5
2B_LT=10^-2
2B_LT=3·10^-3
0
5
10
15
20
Es/N0 [dB]

CYCLE SLIPPING

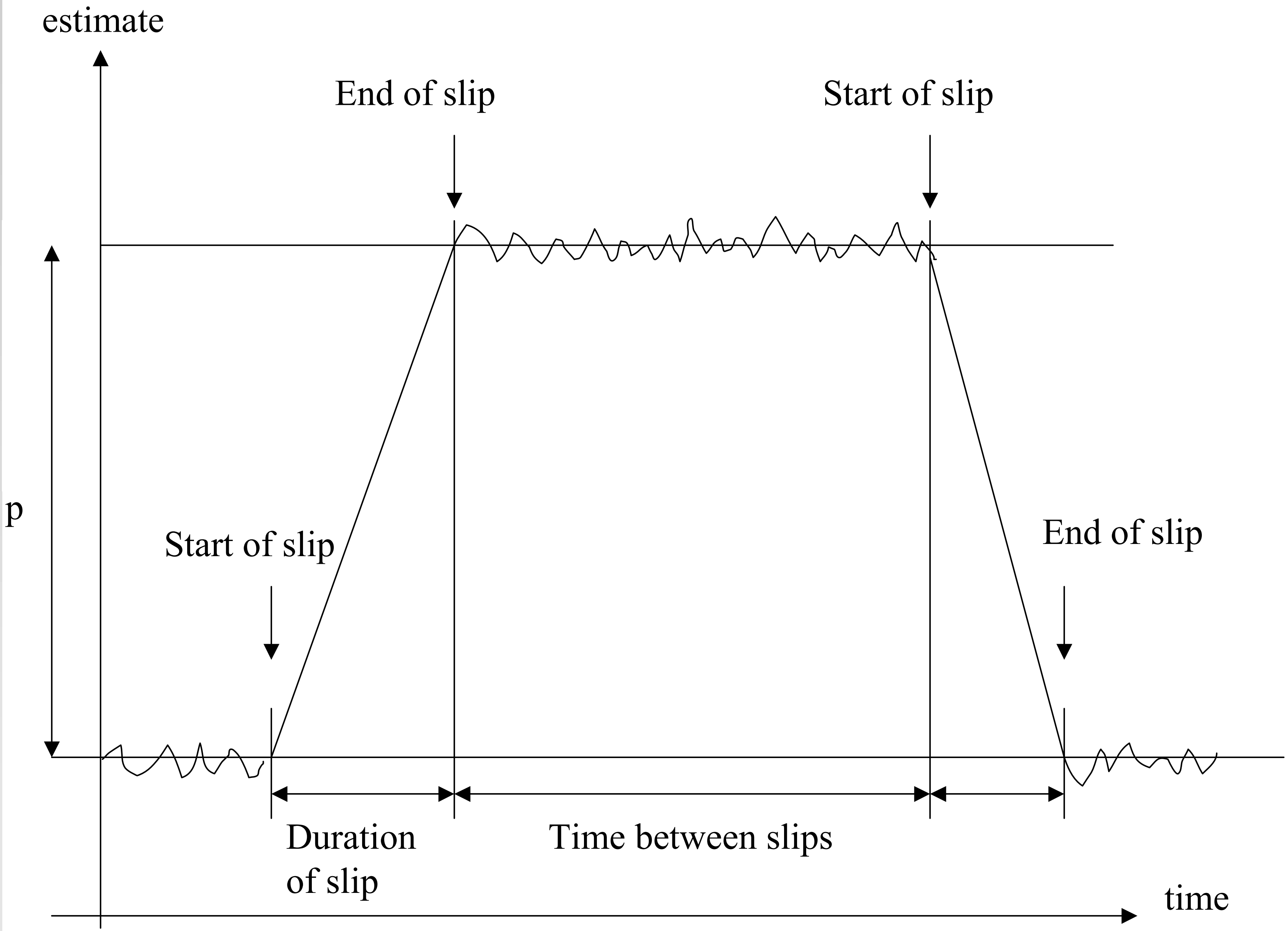
CARRIER

With signal constellation invariant under rotation of angle p, the carrier synchronizer cannot distinguish between an angle θ and an angle θ+kp, with k= ±1, ±2,... $\longrightarrow$ Different stable operating points spaced by p:

- M-PAM p=π
- M^2-QAM p= $\pi/2$
- M-PSK p= $2\pi/M$

TIMING

Because the received signal is cyclostationary with period T, the symbol synchronizer cannot distinguish between the normalized delay ε and ε+kp, with k= ±1, ±2,... and p=1. $\longrightarrow$ Different stable operating points spaced by p:


estimate
End of slip
Start of slip
p
Start of slip
End of slip
Duration
of slip
Time between slips
time

CYCLE SLIPS IN FEEDBACK SYNCHRONIZERS

FOKKER-PLANCK THEORY: for continuos-time systems

When the loop bandwidth of the discrete-time synchronizer is much smaller than the rate at which the carrier phase or timing estimates are updated, a continuous-time synchronizer model can be derived, such that the estimates resulting from the discrete-time synchronizer model are samples of the estimate resulting from the continuous-time synchronizer model

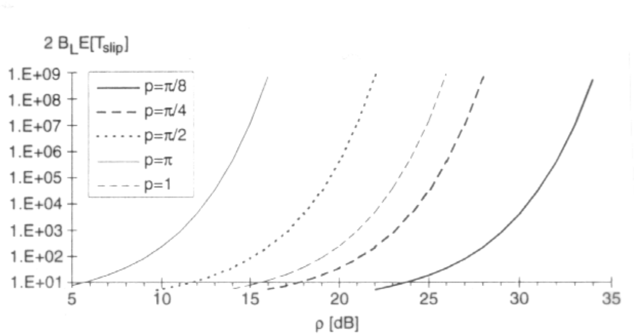


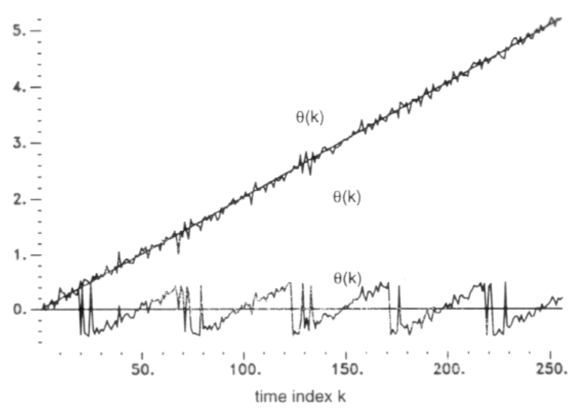
Table 6-1 Dependence of Mean Time Between Slips on Loop Bandwidth for $p = \pi/2$

$B_L T$	ρ [dB]	$E[T_{slip}]/T$	$E[T_{slip}]@T = 1\mu s$
1×10^{-3}	25	1.2×10^{-20}	3.8 million years
3×10^{-3}	20.2	1.3×10^{-8}	1.1 minutes
1×10^{-2}	15	4.1×10^{-3}	4.1 ms
3×10^{-2}	10.2	9.7×10^{-1}	97 μs
1×10^{-1}	5	9.8×10^{0}	9.8 μs

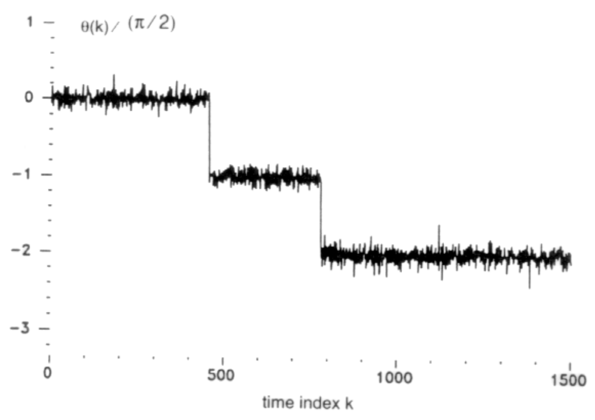
Normalized mean time between Slips

Dependence of mean time between Slips on loop bandwidth for p=π/2

CYCLE SLIPS IN FEEDFORWARD SYNCHRONIZERS



Carrier phase estimate for 4-PSK



Cycle slipping at Post-Processor output (4-PSK)