State Feedback Controller and Luenberger Observer Design for a Two-Wheeled Mobile Robot Following a Path

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The aim of this project is to design and implement a control algorithm and a Luenberger observer in order that a mobile two-wheeled robot follows a path. In order to do it, we will study the observability and controllability of the system. Simulations of the tracking system will be done and then introduced to the robot's code and tested in a simple circuit.

Keywords: Control, Observer, Two-wheeled robot.

I. INTRODUCTION

A two-wheeled robot is a simple robot formed by a chassis, an electronic and a mechanical part. It is schematically composed of two independent actuated wheels on a common axle whose direction is linked to the robot chassis, and one passively orientable wheel, which is not controlled and serves for sustentation purposes. The robot has a sensor at the front which is able to detect a color change of the path, so it is able to search for the line until it finds it and remains over it.

First of all we will study the control problem of the robot following a path with a certain velocity.

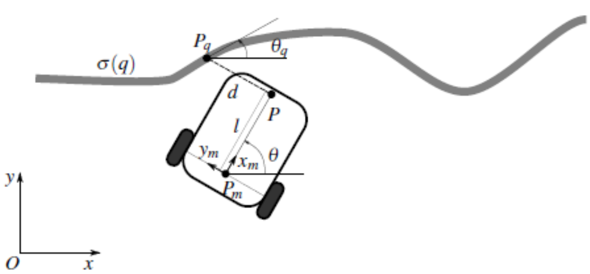

Fig. 1. Two-wheel differential drive mobile robot

In order to do that, we will study the stable case of going forward and with any curvature and then we will focus on the simplest case of cero curvature.

II. MATHEMATICAL MODEL OF A TWO-WHEELED MOBILE ROBOT PROBLEM FORMULATION

The kinematic model for the robot with respect to point P_m is given by

$$\begin{cases} \dot{x} = \cos\theta\, u_1 \\ \dot{y} = \sin\theta\, u_1 \\ \dot{\theta} = u_2 \end{cases}$$

where we defined u_1 and u_2 as

$$u_1 = \frac{r}{2}(w_l + w_r) \qquad u_2 = \frac{r}{2R}(w_l - w_r)$$

being w_l and w_r the angular velocities of the left and right wheels, respectively.

Defining a new angle $\theta_e = \theta - \theta_q$ where θ_q is the angle of the path at point P_q, see figure 1, we get the system

$$\begin{cases} \dot{d} = lu_2 - \tan\theta_e\,(u_1 + du_2) \\ \dot{q} = \dfrac{u_1 + du_2}{\cos\theta_e} \\ \dot{\theta_e} = -u_2 - \dfrac{c}{\cos\theta_e}(u_1 + du_2) \end{cases}$$

Where, d is the distance between the P point and the trajectory, $c(x)$ is curvature of the path and $\dot{q}$ is the velocity of the robot. R is the distance between the two wheels and r is the wheels' radius. See details in [1], [3] and [4].

The control objective is that the robot follows the line, i.e. $d^* = 0$ and $\dot{q^*} = v$. At the equilibrium, this requires u_1, u_2 to be

$$u_1{}^* = v\sqrt{1 - l^2c^2}$$
$$u_2{}^* = -cv$$
$$\theta_e{}^* = \arcsin(-cl)$$

The linearized system in the working points becomes

$$\begin{pmatrix} \dot{d} \\ \dot{\theta}_e \end{pmatrix} = \frac{v}{\alpha}\begin{pmatrix} -lc^2 & -1 \\ c^2 & lc^2 \end{pmatrix}\begin{pmatrix} d \\ \theta_e \end{pmatrix} + \frac{1}{\alpha}\begin{pmatrix} lc \\ -c \end{pmatrix}\tilde{u}_1 + \begin{pmatrix} l \\ -1 \end{pmatrix}\tilde{u}_2$$

Where we defined $\alpha = \sqrt{1 - l^2c^2}$, $\tilde{u}_1 = u_1 - {u_1}^*$ and $\tilde{u}_2 = u_2 - {u_2}^*$.

From now on, we will set $u_1 = {u_1}^*$, since u_1 is our input variable, obtaining the following system

$$\begin{pmatrix} \dot{d} \\ \dot{\theta}_e \end{pmatrix} = \frac{v}{\alpha}\begin{pmatrix} -lc^2 & -1 \\ c^2 & lc^2 \end{pmatrix}\begin{pmatrix} d \\ \theta_e \end{pmatrix} + \begin{pmatrix} l \\ -1 \end{pmatrix}\tilde{u}_2$$

The previous dynamics can be written as a transfer function of the input/output system

$$D(s) = \frac{ls + \alpha v}{s^2 + v^2c^2} U_2(s)$$

I f we consider the case: $c = 0$ so $\alpha = 1$, that leads to

$$D(s) = \frac{ls + v}{s^2} U_2(s)$$

III. OUTPUT TRACKING OBSERVER DESIGN

We analyse the observability condition of the system with the output $y = d$. We obtain the following observability matrix

$$W_o = \begin{pmatrix} 1 & 0 \\ \frac{-lvc^2}{\alpha} & \frac{-v}{\alpha} \end{pmatrix}$$

W_o is full rank since $\alpha > 0$ and $v \neq 0$ and therefore our system will be observable. Then we propose a Luenberger observer of the form

$$\begin{pmatrix} \dot{\tilde{d}} \\ \dot{\tilde{\theta}}_e \end{pmatrix} = \frac{v}{\alpha}\begin{pmatrix} -lc^2 & -1 \\ c^2 & lc^2 \end{pmatrix}\begin{pmatrix} \tilde{d} \\ \tilde{\theta}_e \end{pmatrix} + \begin{pmatrix} l \\ -1 \end{pmatrix}\tilde{u}_2 + \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}(d - \tilde{d})$$

Which considering a straight line (c=0) we get

$$\begin{pmatrix} \dot{\tilde{d}} \\ \dot{\tilde{\theta}}_e \end{pmatrix} = \begin{pmatrix} 0 & -v \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \tilde{d} \\ \tilde{\theta}_e \end{pmatrix} + \begin{pmatrix} l \\ -1 \end{pmatrix}\tilde{u}_2 + \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}(d - \tilde{d})$$

That yields the following state matrix

$$A_O = \begin{pmatrix} -L_1 & -v \\ -L_2 & 0 \end{pmatrix}$$

With eigenvalues

$$\lambda = \frac{-L_1 \pm \sqrt{{L_1}^2 + 4vL_2}}{2}$$

Then the observer will be stable if ${L_1}^2 + 4vL_2 < 0$ and $L_1 > 0$.

IV. OUTPUT TRACKING CONTROLLER DESIGN

The next step is designing the controller of the robot. We propose a state-feedback controller

$$u_2 = -k_d\tilde{d} - k_\theta\tilde{\theta}_e$$

So our system in closed loop becomes

$$\begin{pmatrix} \dot{\tilde{d}} \\ \dot{\tilde{\theta}}_e \end{pmatrix} = \left(\begin{pmatrix} 0 & -v \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} l \\ -1 \end{pmatrix}\begin{pmatrix} k_d & k_\theta \end{pmatrix}\right)\begin{pmatrix} \tilde{d} \\ \tilde{\theta}_e \end{pmatrix}$$

Yielding in the following matrix state

$$A = \begin{pmatrix} -lk_d & -v - lk_\theta \\ k_d & k_\theta \end{pmatrix}$$

whose eigenvalues are

$$\lambda = \frac{-(lk_d - k_\theta) \pm \sqrt{(lk_d - k_\theta)^2 - 4vk_d}}{2}$$

We want the system to be stable, therefore the following conditions must apply:

$$k_d l > k_\theta$$
$$4vk_d > (lk_d - k_\theta)^2$$

Obtaining two poles with imaginary parts that we set such that the stablishing time is 0.1s, time enough for the robot to do the control action.

V. ASSEMBLING THE ROBOT

Before implementing the observer and controller, we needed a mobile robot to be built. There were already three chassis done, but all the connections were needed. So the first laboratory days, we had to weld the connections with tin. The final result of the robot we used to test our controller design is shown in Figure 2.

The robot has a line sensor LRE-F22 (a), two controlled wheels at the front (b), plus a free directional wheel (c) at the back; a driver L298N (d) for the DC motors; two ultrasound HC-SR04 sensors (e), and a chip ESP8266 (f) for the WiFi communications with the PC. The alimentation is done through 4 rechargeable batteries (g) A that provide 5V of voltage. There are several components that need to be alimented at 3.3V so a regulator was also needed and a few filters so as to avoid peaks. In order to do all of this, a discovery is used, with a STM32F407 microcontroller (h), that also provides 5V alimentation. There were also different filters for the components.

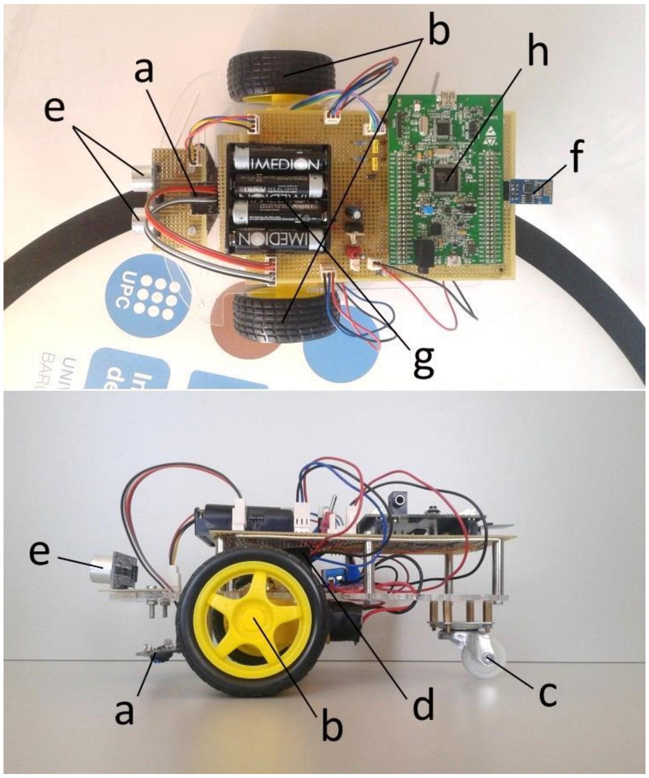

Fig. 2. Top and front view of the mobile robot

VI. EXPERIMENTAL RESULTS

The chosen circuit where we made the robot drive along is a simple closed circuit composed by two large straight lines and the curves that connect them.

After implementing the observer correctly and solving the problems that arised, we could take the data when the robot does one and a half lap to the circuit, and see the graphics of the observed variables in figure 3. In them, we can distinguish when the robot takes the curves because sudden peaks appear in the graphs and the error is bigger.

We can see that our observer works quite well and that the error on the distance is small. We can even see that in the straight parts of the circuit, the estimated angle is almost zero.

There is also needed to point out that the controller is implemented only when the curvature is c = 0 and, even though the error is bigger on the curves, it keeps working quite well.

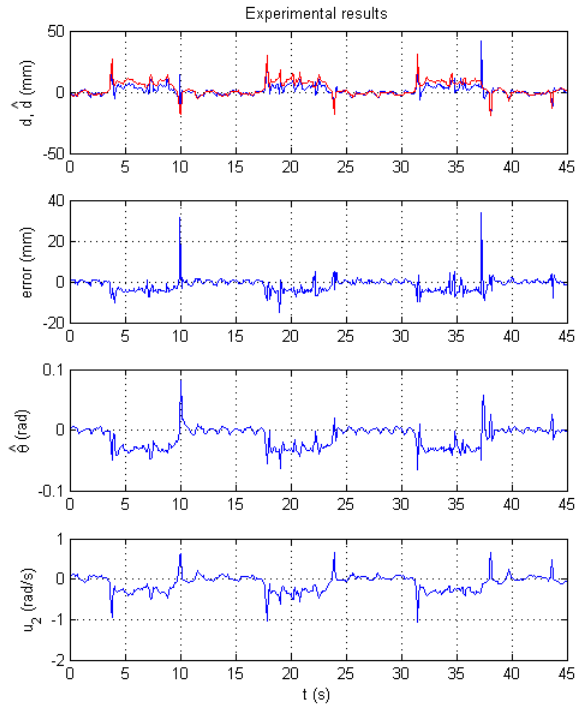

Fig 3. From top to bottom: (1) Comparison between the distance measured, in red, and observed, in blue. (2) Error between measured and observed distance. (3) Observed angle. (4) Control action u_2.

VII. SIMULATIONS OF A TWO-WHEELED ROBOT FOLLOWING A PATH BACKWARDS

We did not have enough time to test our robot going backwards, but we were able to do some simulations of the behaviour of d and θ_e variables when the robot has negative velocity. The simulations in figure 4 prove that the observer designed works properly and we could implement the backwards controller using the observed angle θ_e as a parameter.

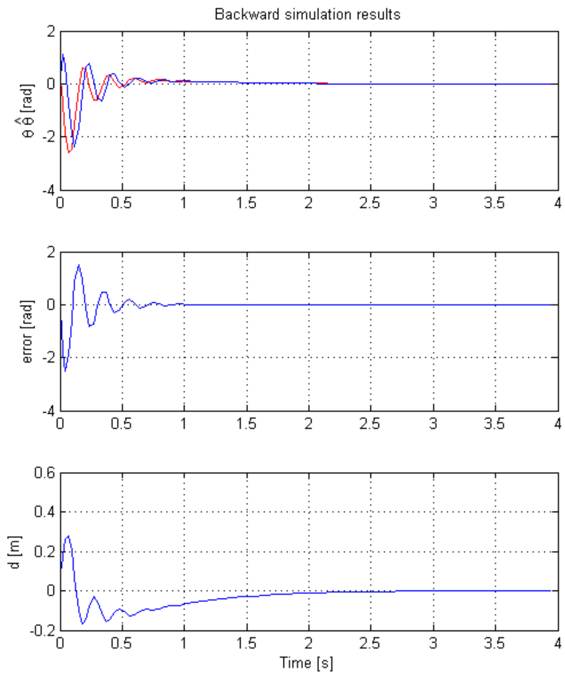

Fig 4. From top to bottom: (1) Comparison between observed (in red) and simulated angle. (2) Error between observed and simulated angle. (3) Behaviour of the distance

VIII. CONCLUSION

After a thorough study of the stability, controllability and observability of the two-wheeled robot dynamic system we finally were able to achieve our goal: the robot following a path.

We have considered the robot following a straight path, in other words, with zero curvature. Nevertheless, the model has been designed keeping in mind the curvature of the line. That means it is possible to use our model as the starting point to design an adaptive controller which would be able to correct the value of the curvature instantly while the robot drives through a complex path.

Finally, we would like to point out that it was also our aim to make the robot go backwards. However, because of several implementation difficulties and the lack of time it was impossible to accomplish it.

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REFERENCES

[1] A. Dòria-Cerezo, D. Biel and V. Repecho, Sliding mode control of a unicycle type differential-drive mobile robot following a path, Tech2017.
[2] P. Morin and C. Samson. Motion control of wheeled mobile robots. In B. Siciliano and O. Khatib, editors, Handbooks of Robotics, chapter 34, pages 779–826. Springer, 2008.
[3] I. Prats-Martinho, Control design and implementation for a line tracker vehicle, TFG, Universitat Politècnica de Catalunya, 2016.
[4] A. Costa-Ruiz, Design of controllers and its implementation for a line tracker vehicle TFG, Universitat Politècnica de Catalunya, 2017.
[5] A. Riera-Seguí, Disseny i implementació d'un sistema de comunicacions WiFi per a una xarxa de vehicles autònoms TFG, Universitat Politècnica de Catalunya, 2016.