TAULA DE PRIMITIVES

FORMES ELEMENTALS.

1. $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C \quad (\text{si } n \neq -1)$
2. $\int (f(x))^n f'(x) \, dx = \dfrac{(f(x))^{n+1}}{n+1} + C \quad (\text{si } n = -1)$
3. $\int \dfrac{dx}{x} = \ln|x| + C$
4. $\int \dfrac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$
5. $\int e^x \, dx = e^x + C$
6. $\int a^x \, dx = \dfrac{a^x}{\ln a} + C$
7. $\int \sin x \, dx = -\cos x + C$
8. $\int \cos x \, dx = \sin x + C$
9. $\int \tan x \, dx = -\ln|\cos x| + C$
10. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \quad \arcsin \dfrac{x}{a} + C = \quad - \arccos \dfrac{x}{a} + C'$
11. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a} \arctan \dfrac{x}{a} + C$
12. $\int \dfrac{dx}{\sin ax} = \dfrac{1}{a} \ln \left| \tan a \dfrac{x}{2} \right| + C$
13. $\int \dfrac{dx}{\cos ax} = \dfrac{1}{a} \ln \left| \dfrac{1}{\cos ax} + \tan ax \right| + C$
14. $\int \dfrac{dx}{\sin^2 ax} = \dfrac{-1}{a \tan ax} + C$
15. $\int \dfrac{dx}{\cos^2 ax} = \dfrac{1}{a} \tan ax + C$
16. $\int \arcsin ax \, dx = x \arcsin ax + \dfrac{1}{a} \sqrt{1 - a^2 x^2} + C$
17. $\int \arccos ax \, dx = x \arccos ax - \dfrac{1}{a} \sqrt{1 - a^2 x^2} + C$
18. $\int \arctan ax \, dx = x \arctan ax - \dfrac{1}{2a} \ln(1 + a^2 x^2) + C$

FORMES TRIGONOMÈTRIQUES

19. $\int \sin^2 ax \, dx = \dfrac{x}{2} - \dfrac{\sin 2ax}{4a} + C$
20. $\int \cos^2 ax \, dx = \dfrac{x}{2} + \dfrac{\sin 2ax}{4a} + C$
21. $\int \tan^2 ax \, dx = \dfrac{1}{a} \tan ax - x + C$

22. $\int \sin ax \cos bx \ dx = -\dfrac{\cos(a+b)x}{2(a+b)} - \dfrac{\cos(a-b)x}{2(a-b)} + C \ (\text{si } a^2 \neq b^2)$

23. $\int \sin ax \sin bx \ dx = \dfrac{\sin(a-b)x}{2(a-b)} - \dfrac{\sin(a+b)x}{2(a+b)} + C \ (\text{si } a^2 \neq b^2)$

24. $\int \cos ax \cos bx \ dx = \dfrac{\sin(a-b)x}{2(a-b)} + \dfrac{\sin(a+b)x}{2(a+b)} + C \ (\text{si } a^2 \neq b^2)$

25. $\int \sin ax \cos ax \ dx = -\dfrac{\cos 2ax}{4a} + C = \dfrac{\sin^2 ax}{2a} + C' = -\dfrac{\cos^2 ax}{2a} + C''$

26. $\int \sin^n ax \ dx = \dfrac{-\sin^{n-1} ax \quad \cos ax}{na} + \dfrac{n-1}{n} \int \sin^{n-2} ax \ dx$

27. $\int \cos^n axdx = \dfrac{\cos^{n-1} ax \quad \sin ax}{na} + \dfrac{n-1}{n} \int \cos^{n-2} ax \ dx$

28. $\int x \sin ax \ dx = \dfrac{1}{a^2} \sin ax - \dfrac{x}{a} \cos ax + C$

29. $\int x \cos ax \ dx = \dfrac{1}{a^2} \cos ax + \dfrac{x}{a} \sin ax + C$

30. $\int x^n \sin ax \ dx = -\dfrac{x^n}{a} \cos ax + \dfrac{n}{a} \int x^{n-1} \cos ax \ dx$

31. $\int x^n \cos ax \ dx = \dfrac{x^n}{a} \sin ax - \dfrac{n}{a} \int x^{n-1} \sin ax \ dx$

32. $\int \sin^n ax \cos^m ax \ dx = -\dfrac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} +$

$+\dfrac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \ (\text{si } m+n \neq 0)$

33. $\int \sin^n ax \cos^m ax \ dx = \dfrac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} +$

$+\dfrac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \ (\text{si } m+n \neq 0)$

34. $\int \tan^{\,n} ax \ dx = \dfrac{\tan^{\,n-1} ax}{a(n-1)} - \int \tan^{\,n-2} ax \ (\text{si } n \neq 1)$

35. $\int \dfrac{1}{\tan^{\,n} ax} dx = \dfrac{1}{a(n-1)\tan^{\,n-1} ax} - \int \dfrac{1}{\tan^{\,n-2} ax} \ (\text{si } n \neq 1)$

<u>FORMES QUE INCLOUEN EXPONENCIALS I LOGARITMES</u>

36. $\int x e^{ax} \ dx = \dfrac{e^{ax}}{a^2}(ax-1) + C$

37. $\int x^n e^{ax} \ dx = \dfrac{1}{a} x^n e^{ax} - \dfrac{n}{a} \int x^{n-1} e^{ax} \ dx$

38. $\int e^{ax} \sin bx \ dx = \dfrac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + C$

39. $\int e^{ax} \cos bx \ dx = \dfrac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx) + C$

40. $\int \ln ax \ dx = x \ln ax - x + C$

41. $\int x^n \ln ax \ dx = \dfrac{x^{n+1}}{n+1} \ln ax - \dfrac{x^{n+1}}{(n+1)^2} + C, \ \ (\text{si } n \neq -1)$

42. $\int \frac{\ln ax}{x}\ dx = \frac{1}{2}(\ln ax)^2 + C$

FORMES HIPERBÒLIQUES

43. $\int \text{sh } ax\ dx = \frac{1}{a}\text{ch } ax + C$
44. $\int \text{ch } ax\ dx = \frac{1}{a}\text{sh } ax + C$
45. $\int \text{th } ax\ dx = \frac{1}{a}\ln(\text{ch } ax) + C$
46. $\int \text{sh }^2 ax\ dx = \frac{\text{sh } 2ax}{4a} - \frac{x}{2} + C$
47. $\int \text{ch }^2 ax\ dx = \frac{\text{sh } 2ax}{4a} + \frac{x}{2} + C$
48. $\int \frac{1}{\text{sh } ax}\ dx = \frac{1}{a}\ln\left|\text{th } \frac{ax}{2}\right| + C$
49. $\int \frac{1}{\text{ch } ax}\ dx = \frac{1}{a}\arcsin(\text{th } ax) + C$
50. $\int \frac{1}{\text{sh }^2 ax}\ dx = \frac{-1}{a\text{th } ax} + C$
51. $\int \frac{1}{\text{ch }^2 ax}\ dx = \frac{1}{a}\text{th } ax + C$

FORMES QUE INCLOUEN $\sqrt{x^2 \pm a^2}$

52. $\int \sqrt{x^2 \pm a^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
53. $\int \frac{dx}{\sqrt{x^2 \pm a^2}}\ dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
54. $\int \frac{\sqrt{x^2 + a^2}}{x}\ dx = \sqrt{x^2 + a^2} - a\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$
55. $\int \frac{\sqrt{x^2 - a^2}}{x}\ dx = \sqrt{x^2 - a^2} - a\ \text{arcsec } \left|\frac{x}{a}\right| + C$
56. $\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$
57. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec } \left|\frac{x}{a}\right| + C = \frac{1}{a}\text{arccos } \left|\frac{a}{x}\right| + C'$
58. $\int x^2\sqrt{x^2 \pm a^2}\ dx = \frac{x}{8}(2x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
59. $\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\ dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
60. $\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$
61. $\int \frac{\sqrt{x^2 \pm a^2}}{x^2}\ dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

FORMES QUE INCLOUEN $\sqrt{a^2-x^2}$

62. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

63. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{a^4}{8}\arcsin\frac{x}{a} - \frac{1}{8}x\sqrt{a^2-x^2}(a^2-2x^2) + C$

64. $\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

65. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \arcsin\frac{x}{a} + C$

66. $\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = \frac{a^2}{2}\arcsin\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + C$

67. $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

68. $\int \frac{1}{x^2\sqrt{a^2-x^2}}\,dx = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

FORMES QUE INCLOUEN $ax+b$ O B $\sqrt{ax+b}$

69. $\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left(\frac{ax+b}{n+2} - \frac{b}{n+1}\right) + C \quad (\text{si } n \neq -1, -2)$

70. $\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$

71. $\int \frac{x}{(ax+b)^2}\,dx = \frac{1}{a^2}\left(\ln|ax+b| + \frac{b}{ax+b}\right) + C$

72. $\int \left(\sqrt{ax+b}\right)^n\,dx = \frac{2}{a}\frac{\left(\sqrt{ax+b}\right)^{n+2}}{n+2} + C \quad (\text{si } n \neq -2)$

73. $\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{1}{x\sqrt{ax+b}}\,dx$

74. $\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{2}{\sqrt{-b}}\operatorname{arctg}\sqrt{\frac{ax+b}{-b}} + C \ (\text{si } b < 0)$

75. $\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C \ (\text{si } b > 0)$