LASER RADAR SYSTEMS

OPTICAL REMOTE SENSING
(F. Rocadenbosch)

PROBLEM 2: LINK-BUDGET (ELASTIC-RAMAN LIDAR)

Consider an elastic-Raman lidar defined by the following system parameters:

LASER

Quantel (Nd:YAG 2ω)		
Emission wavelength, λ_0	532 nm	
Energy, E	(parameter) mJ	
Pulse-repetition frequency, PRF	(parameter) Hz	
Beam width, w_0	5×10^{-3} m	(*)
Divergence (half-width angle), $\theta_{1/2}$	0.5 mrad	(*)

TELESCOPE

Celestron Schmidt-Cassegrain	
Primary lens diameter, d_p	(parameter) m
Shade diameter, d_{sh}	0.06858 m
Focal length, f	2 m
Transmissivity, T_1	60 %

ELASTIC-RECEIVING CHANNEL

Reception wavelength, λ_0	532 nm

INTERFERENCE FILTER

Bandwidth, $d\lambda_0$	10 nm
Transmissivity, T_2	65 %

PHOTODIODE

APD (EGG C30956E)	
Active area diameter, D_{APD}	3 mm
Multiplication factor, M	(parameter)
Excess-noise factor, F	(parameter)
Dark surface current, I_{ds}	7.64×10^{-8} A
Dark bulk current, I_{db}	3.10×10^{-10} A
Intrinsic responsivity, R_{io}	240 mA/W

SIGNAL-CONDITIONING STAGES

Transimpedance Gain (1st stage), G_t	5750 Ω
Voltage conditioning Gain (2nd stage), G_{ac}	20.3 V/V
Noise-equivalent bandwidth, B	10 MHz
Equivalent input noise (chain input), $\sigma_{th,i}$	5 $\text{pA}\cdot\text{Hz}^{-1/2}$

ACQUISITION SYSTEM

Type: Analog-to-digital recorder	ADC
Sampling frequency, f_s	20 Msps (20×10^6)

RAMAN-RECEIVING CHANNEL

• Reception wavelength, λ_R	607.4 nm

INTERFERENCE FILTER

• Bandwidth, $d\lambda_R$	(parameter) nm
• Transmissivity, T_2’	65 %

PMT (PHOTO-MULTIPLIER TUBE)

(Please note APD-equivalent notation in use)

• Equivalent active area diameter, D	3 mm
• Multiplication factor, M	(parameter)
• Excess-noise factor, F	1.8
• Anode dark current, I_{da} (i.e, $I_{ds}=0$, $I_{db}=I_{da}/M$)	(parameter) nA
• Anode radiant sensitivity, R_i ($R_{io}=R_i/M$)	(parameter) A/W

SIGNAL-CONDITIONING STAGE

• PMT-load resistance, R (i.e, G_t)	50 Ω
• Noise-equivalent bandwidth, B	10 MHz

ACQUISITION SYSTEM (See also “Hamamatsu Appl. Note” in pdf.)

• Type: Photon counter	PC
• Temporal resolution, Δt_{PC}	1 bin
• Sampling (“binning”) period	50 ns/bin

ATMOSPHERE

Aerosol component at λ_0:

• Visibility margin (532 nm), V_M	(parameter) km
• Lidar ratio, $S_M(\lambda_0)=\alpha_{Mie}(\lambda_0)/\beta_{Mie}(\lambda_0)$	(parameter) sr

Aerosol component at λ_R:

• Wavelength-dependency coef., κ=1.8. I.e., consider	$\frac{\alpha_{Mie}(\lambda_R)}{\alpha_{Mie}(\lambda_0)}=\left(\frac{\lambda_R}{\lambda_0}\right)^{-\kappa}$

Molecular components at λ_0, λ_R (Height-dependent, US-standard atmosphere approx.)

• Rayleigh’s extinction, $\alpha_{Ray}(\lambda_i,R)$	km^{-1} with R [km]

$$\alpha_{Ray}(\lambda_0,R) \approx 1.2569\times10^{-2} - 7.7599\times10^{-4}\ R$$
$$\alpha_{Ray}(\lambda_R,R) \approx 7.3219\times10^{-3} - 4.5204\times10^{-4}\ R$$

• Rayleigh’s ratio, $S_R(\lambda)=\alpha_{Ray}(\lambda)/\beta_{Ray}(\lambda)$	$8\pi/3$
• N_2-Raman backscattering cross-section, $d\sigma_{N2}(\pi)/d\Omega$	3.71×10^{-41} [$km^2\cdot sr^{-1}$]
• N_2-molecule number density, $N_{N2}(R)$ (height-dep.)	[km^{-3}] with R [km]

$$N_{N2}(R) \approx 2.1145\times10^{34} - 2.0022\times10^{33}R + 5.4585\times10^{31}R^2$$

Boundary-layer height, R_{PBL}	(parameter) km

Background-radiance component

• Moon’s radiance (full Moon), L_{Moon}	3×10^{-11} $W\cdot cm^{-2}\cdot nm^{-1}\cdot sr^{-1}$
• Solar radiance, L_{Sun} (typ.)	3×10^{-6} $W\cdot cm^{-2}\cdot nm^{-1}\cdot sr^{-1}$

OTHER PARAMETERS

Full-overlap range, R_{ovf}	200 m	
Maximum-range criterion	$SNR(R_{max}) = 1$	
Observation time (to simulate), t_{obs}	$[1/PRF, 10^4]$ s	

PHYSICAL CONSTANTS

Electron charge, q	1.602×10^{-19} C	
Planck's constant, h	6.6262×10^{-34} J·s	
Light speed, c	2.99793×10^{8} m·s^{-1}	
Boltzmann's constant, K	1.38×10^{-23} J·K^{-1}	(*)

(*) Parameter not used.

Questions:

1. Determine the system constant, K [W·km^3].

2. Estimate the received background power for both the elastic and Raman channels ($P_{back,0}$ and $P_{back,R}$, respectively) under (see "day-time/night-time" parameter) operation.

3. Plot both elastic- and Raman-return powers ($P_0(R)$ and $P_R(R)$, respectively). Superimpose $P_{back,0}$ and $P_{back,R}$ plots from question 2 results.

4. Compute, for the elastic and Raman channels, receiver-chain voltage responsivities ($R_{v,0}$ and $R_{v,R}$, respectively), and net voltage responsivities (i.e., including spectral optical losses; $R_{v,0}$' and $R_{v,R}$', respectively).

5. a) Assuming analog detection, plot the elastic range-dependent signal-to-noise ratio, $SNR_0(R)$, at the output of the receiver chain (i.e., voltage ratio) and related shot photo-induced, shot-dark and thermal variances ($\sigma^2_{sh,s}$, $\sigma^2_{sh,d}$, σ^2_{th} in units of [V^2]).

   b) Identify where you have different noise-dominant system-operation modes.

6. a) (See Hamamatsu Appl. Note, Sect.1 and Sect. 3.8.2, pp. 14-15) Assuming, photon counting detection, plot the Raman range-dependent photon-count signal-to-noise ratio, $SNR_R(R)$ (Eq.(2), p.15) and related shot photo-induced, shot-dark and shot-background variances (in units of [counts/bin]).

7. (Raman-channel only). Departing from your results of question (6) above, give a plot of the required observation time (Y-axis) versus range (X-axis) in order to ensure a goal SNR of $SNR_{min} = 2 \times 10^3$ (linear units) at each successive range.

8. Compute and compare photodiode's NEP (NEP_{APD}) and PMT's NEP (NEP_{PMT}).

9. (OPTIONAL) Assume that only elastic- and Raman-return powers are known (question (3)). Plot the maximum lidar range (Y-axis) versus required observation time (X-axis; i.e. consider pulse integration) for both elastic and Raman channels (see R_{max} criterion).

Figs.1-4. ANSWER PLOT FORMAT

Q3. Fig.1. Abscissae range 0 to 15 km; linear-X, log-Y. $P_0(R)$, $P_R(R)$, $P_{back,0}$ and $P_{back,R}$ in the same plot.

Q5 & Q6. Fig.2. Abscissae range 0 to 15 km; linear-X, log-Y. $SNR_0(R)$ and $SNR_R(R)$ superimposed. Fig.3. Abscissae range 0 to 15 km; linear-X, log-Y, 2 subplots: Fig.3(a) Superimposed elastic variances; Fig.3(b) Superimposed Raman variances.

Q7. Fig.4. Abscissae range 0 to 15 km; linear-X, log-Y.

Q9. Fig.5. Abscissae range 10^{-2} to 10^4 s; log-X, log-Y.

COMMENTS AND SUPPORT FORMULAE

In this problem an elastic-backscatter and a Raman lidar channel operate simultaneously at λ_0 and λ_R. The elastic-backscatter lidar equation

$$P_{\lambda_0}(z) = \frac{K_{\lambda_0}}{z^2}\left[\beta_{\lambda_0}^{aer}(z) + \beta_{\lambda_0}^{mol}(z)\right] \times \exp\left\{-2\int_0^z \left[\alpha_{\lambda_0}^{aer}(\xi) + \alpha_{\lambda_0}^{mol}(\xi)\right] d\xi\right\}\xi(z)$$

has already been addressed in Chap.3 with z=R the vertical direction.

In the case of the Raman channel, the reception channel is spectrally tuned to receive backscattered radiation from atmospheric nitrogen molecules. In Chap. 7, we will see that since nitrogen is an atmospheric abundant species, it is of advantage to calibrate the co-operative elastic channel and hence, to obtain independent estimations of both extinction and backscatter atmospheric optical components. In comparison with the elastic-backscatter lidar equation, in which both the optical emission path (i.e., from the laser source to the atmosphere) and return path (i.e., from the atmosphere back to the telescope) were operating at the same wavelength, λ_0, now, in the Raman case, the emission path operates at λ_0 while the return path operates at λ_R. This translates into a two-way path Raman transmittance

$$T(\lambda_0, z)T_R(\lambda_R, z); \quad T(\lambda_i, z) = \exp\left[-\int_0^z \alpha_{\lambda_i}^{mol}(\xi) + \alpha_{\lambda_i}^{aer}(\xi)\right]$$

instead of the well-known two-way path elastic transmittance,

$$T(\lambda_0, z)^2$$

In addition, since the Raman receiver is specifically tuned to receive backscattered radiation from N_2 atmospheric molecules at the Raman-N_2 shifted wavelength (λ_R=607.4 nm), the Raman backscatter coefficient is computed as,

$$\beta_{\lambda_R}(z) = N_R(z)\frac{d\sigma_{\lambda_R}(\pi)}{d\Omega}$$

where $N_{\lambda_R}(z)$ is the nitrogen molecule number density at λ_R, and $d\sigma_{\lambda_R}(\pi)/d\Omega$ is the range-independent nitrogen Raman backscatter cross-section per solid angle unit (see also Chap.2).

Thus, the Raman lidar equation takes the form

$$P_{\lambda_R}(z) = \frac{K_{\lambda_R}}{z^2}\left[N_R(z)\frac{d\sigma_{\lambda_R}(\pi)}{d\Omega}\right] \times \exp\left\{-\int_0^z \left[\alpha_{\lambda_0}^{mol}(\xi) + \alpha_{\lambda_0}^{aer}(\xi) + \alpha_{\lambda_R}^{mol}(\xi) + \alpha_{\lambda_R}^{aer}(\xi)\right] d\xi\right\}\xi(z)$$

where all the variables have already been defined either here or in Chap.3.

In the proposed problem, $N_{\lambda_R}(z), \alpha_{\lambda_i}^{mol}(z), \alpha_{\lambda_i}^{aer}(z)$ magnitudes have been approximated from a US-standard atmosphere model. Note, all these magnitudes progressively decrease with height in the troposphere.

Computation of the Raman-channel range-dependent signal-to-noise ratio is completely analogous to the link-budged relations presented in Chap.4 when Analog Detection is used. Yet, in this problem (as well as in practice), Photon Counting Detection is used. Please refer to Sect. 3.8.2, pp. 14-15 of Hamamatsu’s Application Note and use Eq.(2).

CATEGORY	**PARAMETER**	**VALUE (CODE 0)**	**VALUE (CODE 1)**	**VALUE (CODE 2)**	**VALUE (CODE 3)**
LASER	Energy	160	300	100	150
	PRF	20	40	100	30
TELESCOPE	Primary lens diameter	0,2032	0,5	0,4	0,3
ELASTIC CHANNEL	PHOTODIODE, Multiplication factor	150	150	400	150
	PHOTODIODE, Excess-noise factor	4,5	4,5	4,5	4,5
RAMAN CHANNEL	INTERFERENCE FILTER, bandwidth	3	0,5	3	3
	PMT, Multiplication factor	3,00E+06	3,00E+06	3,00E+06	1,00E+06
	Anode dark current	1	1	1	1
	Anode radiant sensitivity	3,00E+04	3,00E+04	6,00E+04	3,00E+04
ATMOSPHERE	Visibility margin	39,12	3,912	39,12	3,912
	Lidar ratio, SM	25	25	25	25
	Boundary-layer height	3	3	3	3
	System operation (question 2)	night-time	day-time	day-time	night-time

NOTE: **You will be assigned a CODE NO. to solve this problem**

VALUE (CODE 4)	VALUE (CODE 5)	VALUE (CODE 6)	VALUE (CODE 7)	VALUE (CODE 8)	VALUE (CODE 9)	VALUE (CODE 10)
300	150	100	50	300	50	200
20	30	100	100	20	100	20
0,4	0,3	0,4	0,3	0,5	0,3	0,3
300	150	150	400	400	400	400
4,5	4,5	4,5	4,5	6	4,5	6
10	3	0,5	0,5	1	3	3
3,00E+06	1,00E+06	3,00E+06	3,00E+06	3,00E+06	3,00E+06	3,00E+06
5	1	1	0,1	1	1	1
3,00E+04	3,00E+04	3,00E+04	3,00E+04	3,00E+04	3,00E+04	3,00E+04
39,12	3,912	39,12	39,12	3,912	39,12	3,912
25	25	25	50	25	25	25
3	3	2	3	3	2	2
day-time	night-time	night-time	night-time	night-time	day-time	night-time

VALUE (CODE 11)	VALUE (CODE 12)	UNITS
500	150	mJ
10	20	Hz
0,5	0,2	m
400	150	no units
4,5	4,5	no units
0,5	3	nm
3,00E+06	3,00E+06	no units
0,5	1	nA
3,00E+04	3,00E+04	A/W
3,912	39,12	km
50	25	sr
3	3	km
night-time	night-time	