Exercises Unit 1. MATLAB Fundamentals

Working period: First and second weeks
Deadline: 3 March 2013

Upload only one file, `my_name_E1.pdf`, containing the solutions for the following exercises. Include both the MATLAB commands used and the results with comments. See the sample file (`exercises_template.pdf` available in Moodle).

1. Basic MATLAB

Exercise 1. Vector operations

The moment of inertia of a sector of a circle is

$$I = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

where r is the radius of the circle in m. Determine I when r is 1.5 cm, 2 cm, 2.5 cm and 3 cm. Store the result in a vector (Function: ".").

Exercise 2. Matrix operations

1) Generate a 3×2 matrix of 1s (function `ones`).

2) Enter the following data: $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $\mathbf{M} = \begin{bmatrix} 1 & 339 \\ 0 & 1 \end{bmatrix}$.

3) Find $\mathbf{C} = \mathbf{A}^T$ (function: ').

4) Obtain the sub-matrix $\mathbf{A}_1$ defined as the 2nd and 3rd columns of **A**. Obtain the sub-matrix $\mathbf{A}_2$ defined as the 1st and 2nd rows and the 1st and 2nd columns of matrix **A**.

5) Compute $\mathbf{C} = \mathbf{A}^{-1}$ (function `inv`).

6) Find the rank, determinant, eigenvalues and <u>eigenvectors</u> of matrix **A** (functions `rank`, `det` and `eig`).

7) Find the (column) eigenvector corresponding to eigenvalue $\lambda = -1.0766$ of matrix **A** and store it in the variable u. (Functions `eig` and `()`).

8) Find the singular values and condition number of matrices **A** and **M**. Which one is closer to singularity? (Functions `svd`, `cond`).

9) Briefly explain the difference between `A.^2` and `A^2`.

Exercise 3. Solution of equation systems

1) Solve the following equation system by using function **`inv`**.

$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= 12 \\ 4x_1 - 5x_2 + 6x_3 &= 10 \\ 7x_1 + 8x_2 + 9x_3 &= 13 \end{aligned}\right\}$$

2) To solve an equation system like the following one, you can use the pseudoinverse matrix. That is, the system

$$\left.\begin{aligned} -0.4x_1 - 1.1x_2 &= 0.95 \\ -1.6x_1 + 1.2x_2 &= 0.23 \\ 0.12x_1 + 1.2x_2 &= 0.61 \\ 0.3x_1 - 0.1x_2 &= 0.48 \end{aligned}\right\}$$

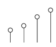
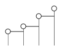
can be written in a matrix form $\mathbf{Hx} = \mathbf{y}$. The optimal solution (in the least-squares sense) is $\mathbf{x} = \mathbf{H}^{\perp}\mathbf{y}$, where $\mathbf{H}^{\perp} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T$ is the pseudoinverse matrix of **H**. Find x_1, x_2 by using function **`pinv`**.

Exercise 4. Simple plots

1) Generate a vector `x` containing values between 0 and 4π and equally spaced by π/10 (function "**`:`**"). Plot the exponential function e^x in such an interval (functions **`exp`** and **`plot`**). Label the plot (functions **`xlabel`**, **`ylabel`** and **`title`**).

2) Repeat **1)** but now generate a vector `x` with first value 0 and final value 1 (function **`linspace`**).

3) Plot <u>exactly 4 cycles</u> of $y_1(x) = \sin(3x)$ (functions **`linspace`**, **`pi`**, **`sin`**). Add to this representation the plot corresponding to the function $y_2(x) = e^{-\frac{x}{8}} \cdot \cos(3x)$ (functions: "**`.`**", **`sin`**, **`cos`**, **`plot`**, [**`hold`**]).

4) Generate an arbitrary signal and plot it as follows: (a) discrete sequence (), (b) continuous signal after a zero order hold (ZOH): (), and (c) continuous signal () (functions **`stem`**, **`stairs`**, **`plot`**). Plot the three representations in the same figure window by using the function **`subplot`**.

Exercise 5. Evaluation and representation of polynomials

Consider the following normalized Butterworth polynomial (obtained using **`buttap`** functions and **`zp2tf`**):

$$p(x) = x^5 + 3.2361x^4 + 5.2361x^3 + 5.2361x^2 + 3.2361x + 1$$

1) Plot $p(x)$ for x varying between -2 and 2 (functions **`linspace`**, **`polyval`**, **`plot`**).

2) Find the polynomial roots and plot them in the complex plane. Verify that they are on a semicircle of radius 1. Functions **`roots`**, **`plot`**, **`axis`**.
3) Optional: Repeat for another polynomial, for example, Bessel (**`besselap`**), Chebychev (**`cheb1ap`**) and Cauer (**`ellipap`**).

Exercise 6. Loading variables from Excel
Choose one of the **`telefonica.xls`** or **`ibex.xls`** files available in the course intranet (it is also possible to use another time series, financial, weather, etc.).

1) Execute the function **`xlsread`** for e.g. the file **`telefonica.xls`** and open the *Array Editor* to see the loaded variables.
2) Save in a variable the column corresponding to the closing prize.
3) Plot these values. Optional: Label the tick lines of the x-axis using the dates (functions **`datenum`**, **`datetick`**).

Exercise 7. Polynomial fit (linear regression) of an experimental relationship
The following measurements have been made to calibrate a measuring instrument, where "y" is the measurement (indication) of the pattern and "x" is the instrument reading.

y	0	1	2	5	10	15
x	0.5	0.72	0.78	1.21	1.76	2.46

In view of the measurements, we will fit a model with the following expression:
$y = a + bx$

1) Plot the data y(x) (use the **`plot`** function with a discrete line option) and roughly choose the a, b parameters that define a straight line that approximately relates them.
2) Express the vector "y" as a function of parameters $\theta^T = (a \quad b)$. That is, build the system $\mathbf{y} = \mathbf{H}\boldsymbol{\theta}$.
3) Find the regression line using the pseudoinverse (**`pinv`** function) or by using the **`polyfit`**.
4) Plot in the same graph: the points on the table, the approximate line (roughly) and the regression line (functions **`polyval`**, **`plot`**).
5) Assess the quality of the fit using the following criteria:
♣ Calculate $J = \sum e_i^2$ (function **`sum`**) for the "roughly line" and for the optimal regression line.
♣ Compute and plot the autocorrelation of the error $R_e(m)$ for both lines (functions **`xcorr`**, **`linspace`**, **`stem`**).

2. Toolboxes

Exercise 8. Complex variable functions
(This exercise uses some functions from the *Control Systems Toolbox*. If you do not have this toolbox, you can skip this exercise.)

1) Given the complex variable function $G(s)=\frac{3}{s(s+1)(s+3)}$, find the modulus and argument when the complex variable "s" takes the following values *s*=0, *j*, 2*j*, 5*j*, and ∞*j* (functions **`polyval`** or **`freqs`**, **`abs`**, **`angle`**).

2) Plot the Bode diagram for the frequency response $G(s)\big|_{s=j\omega}$ (function **`bode`**, [**`logspace`**]).

3) Obtain the polar plot (representation in Nyquist plane) for the frequency response $G(s)\big|_{s=j\omega}$ (functions **`logspace`** and **`nyquist`**). Be careful with the specification of the frequencies vector (we want to see in detail the frequency response near the origin of the Nyquist plane).

Exercise 9. Time response

(This exercise uses some functions from the *Control Systems Toolbox*. If you do not have this toolbox, you can skip this exercise.)

1) Plot the impulse response for the system $H(s)=\frac{2}{s^2+0.2s+1}$ (function **`impulse`**).

2) Plot the step response for the system $H(s)=\frac{2}{s^2+0.2s+1}$ (function **`step`**). What is the peak value? And the establishment time value? Use the right mouse button over the figure to obtain these two values.

Exercise 10. Power Spectral Density

(This exercise uses some functions from the *Signal Processing Toolbox*. If you do not have this toolbox, you can skip this exercise.)

Consider Example 5 in the notes for Unit 1. For the square signal case, repeat the example for other sampling frequencies and other duty cycle values to see how the *sinc* function changes and how the aliasing affects the PSD.