Excerpt from an exam

Consider the vector space $\mathbb{R}^2$ with its standard basis B, and we consider the basis $B' = (u_1, u_2)$

$u_1 = (1,2)|_B$ and $u_2 = (1,1)|_B$.

Consider the vector $v = (-2,-1)|_B$

Plot basis B

Plot basis B'

Plot $v|_B$

Plot $v|_{B'}$

What do you observe?

<u>Answer:</u> $v|_{B'} = D^{-1} v|_B$

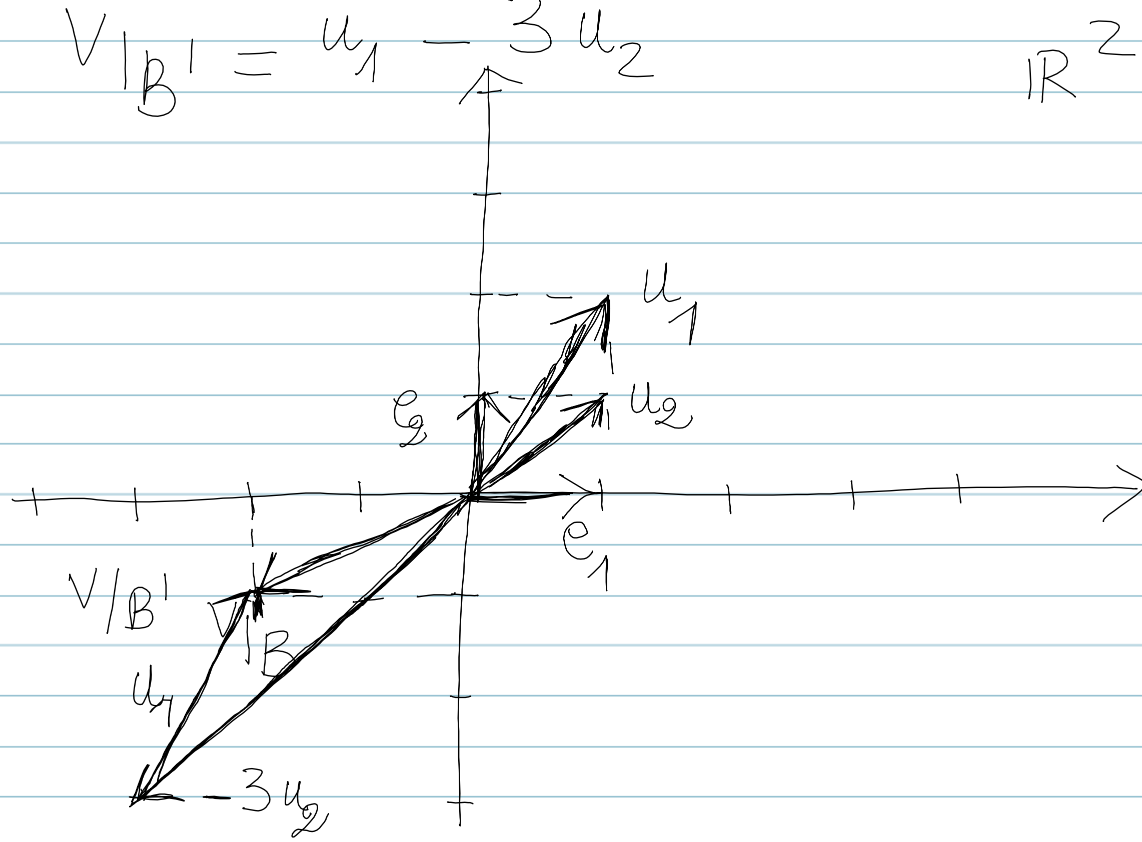
$$D = (u_1 \; u_2) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

Thus $$D^{-1} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$V|_{B'} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$V|_{B'} = (1, -3)|_{B'}$

$V|_{B'} = u_1 - 3u_2$



$B = (e_1 \quad e_2)$ with

$e_1 = (1, 0)|_B$

$e_2 = (0, 1)|_B$

$V|_{B'}$ and $V|_B$ are the same vector.

Student's answer

$v|_{B'} = (1, -3)|_{B'}$