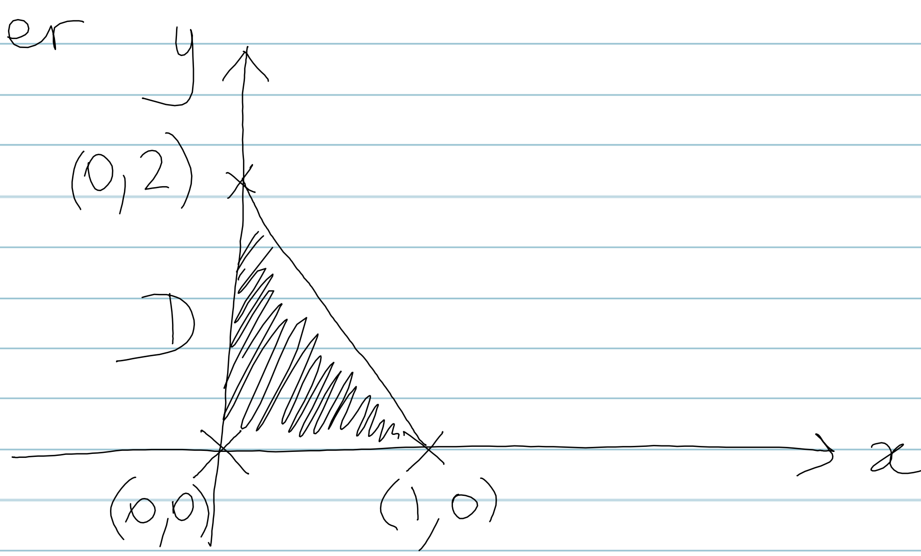
Excerpt from an exam

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x, y) = x + y$ $\forall (x,y) \in \mathbb{R}^2$

let D be the triangle defined by points $(0,0)$; $(1,0)$; $(0,2)$

Find $\iint_D f(x,y)\,dx\,dy$

Answer

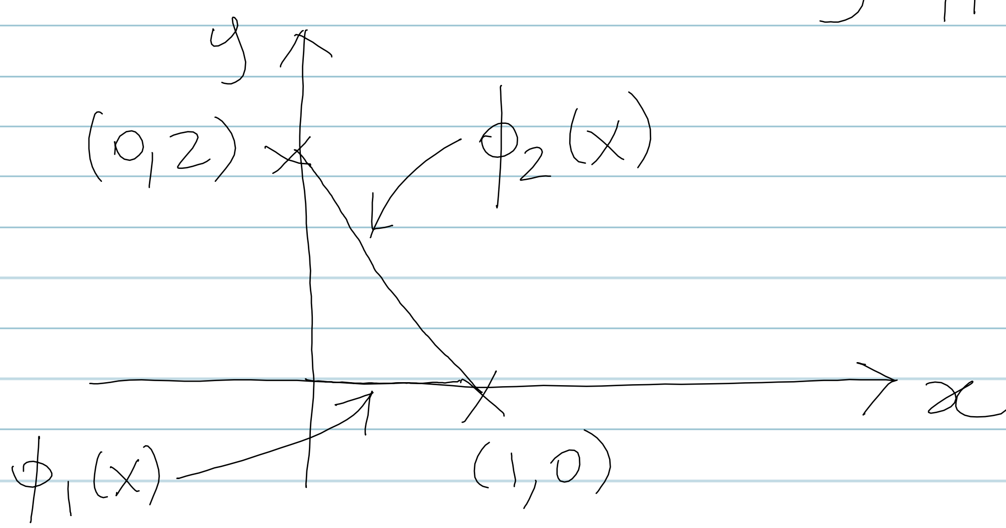


The equation of the line that connects points $(0,2)$ and $(1,0)$ is $y = ax + b$

We have
$$2 = a\cdot 0 + b \quad (\text{at point } (0,2))$$
and $$0 = a\cdot 1 + b \quad (\text{at point } (1,0))$$

These equations give: $b = 2$
$a = -2$

Domain D has the form used in Fubini's theorem. It is upper limited by function $\phi_2(x) = -2x+2$ and is lower limited by $\phi_1(x) = 0$

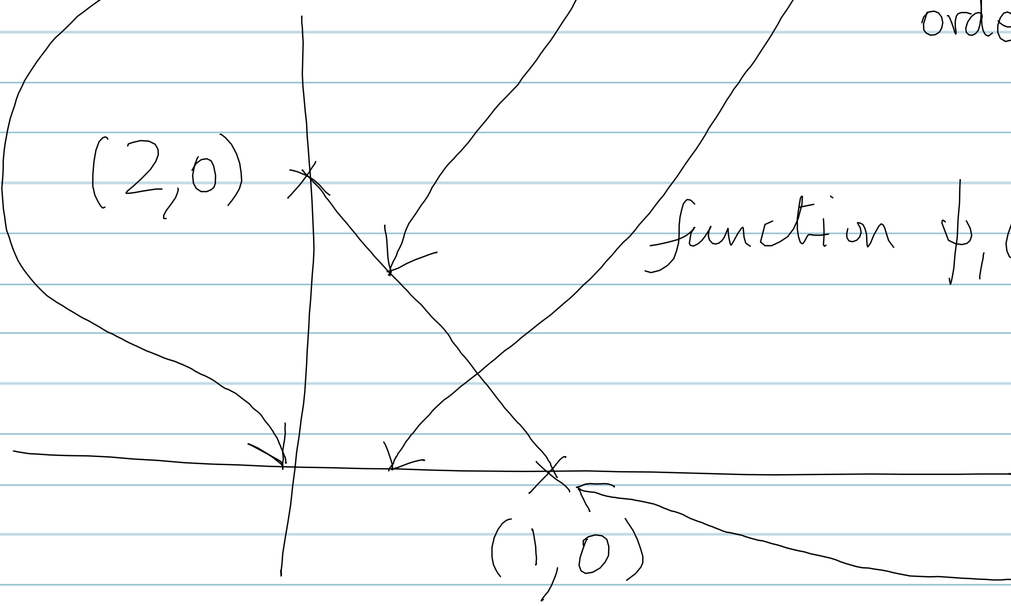


So using Fubini's theorem we obtain

order not important

$$\iint_D f(x,y)\, dx\,dy = \int_0^1 \int_0^{-2x+2} x+y \, dy\, dx$$

in this order



$$= \int_0^1 \left[xy + \frac{y^2}{2} \right]_0^{-2x+2} dx$$

$$= \int_0^1 x(-2x+2) + \frac{(-2x+2)^2}{2} \, dx$$

$$= \int_0^1 -2x + 2\,dx = \left[-x^2 + 2x\right]_0^1 = 1$$

Student's answer

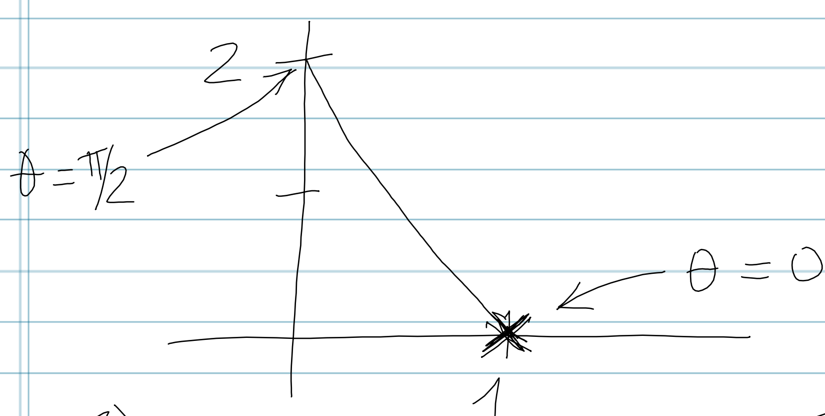
$x = r\cos\theta$

$y = r\sin\theta$

$$= \iint_{\varphi^{-1}} r\cos\theta + r\sin\theta \, d\theta\, dr$$

$\theta = [0, \pi/2]$; $r \in [1, 2]$

the interval for r is incorrect



Correct: Fix θ

minimum value of r $\to 0$

maximal value of r