Except from an exam

Let $f : \mathbb{R} \to \mathbb{R}$ be a know $C^1$ function such that $f(0)=1$, $f'(0)=1$, and $\forall x \in \mathbb{R}$ we have $f(x)>0$.

Define the following function $h : \mathbb{R}^2 \to \mathbb{R}$ such that $\forall (x,y) \in \mathbb{R}^2$ we have

$$h(x,y) = \left[ f(2y-x) \right]^{x\sin(y)}$$

Find $\frac{\partial h}{\partial x}\left(\pi, \frac{\pi}{2}\right)$.

<u>Answer</u> Observe that

$$h(x,y) = e^{x\sin(y)\ln[f(2y-x)]}$$

Since $f(x) > 0$ $\forall x \in \mathbb{R}$, it follows that $\ln[f(2y-x)]$ exists.

The function ~~f~~ $h$ is differentiable at all point $(x,y)$ as a combination of differentiable functions. Thus $\frac{\partial h}{\partial x}$ exists at all points $(x,y) \in \mathbb{R}^2$.

in rough $\left(e^{l(x)}\right)' = l'(x) \cdot e^{l(x)}$

$$\frac{\partial h}{\partial x}(x,y) = \underbrace{\frac{\partial\left(x\sin(y)\ln\left[f(2y-x)\right]\right)}{\partial x}}_{①} \cdot e^{x\sin(y)\ln[f(2y-x)]}$$

$$① = \sin(y) \cdot \frac{\partial\left(x\ln\left[f(2y-x)\right]\right)}{\partial x}$$

$$= \sin(y) \cdot \left[\ln\left[f(2y-x)\right] + x \cdot \underbrace{\frac{\partial \ln\left[f(2y-x)\right]}{\partial x}}_{②}\right]$$

$$② = \frac{\partial}{\partial x}\left(\ln \circ f(2y-x)\right)$$

rough

$$\left(h_1 \circ h_2(x)\right)'$$

$$= h_1'\left(h_2(x)\right) \cdot h_2'(x)$$

rough

$$h_1(x) = \ln(x)$$

$$h_1'(x) = \ln'(x)$$

$$\frac{\partial h_1}{\partial x}(x) = \frac{\partial}{\partial x} \ln(x)$$

$$= \frac{1}{x}$$

$$② = \frac{1}{f(2y-x)} \cdot \underbrace{\frac{\partial}{\partial x}\left( f(2y-x) \right)}_{③}$$

~~instead of~~ $\cancel{\frac{\partial}{\partial x}\left( f(2y-x) \right)}$

$\cancel{f'(2y-x)}$

③ is the derivative of the composition of two functions

$$③ = \frac{\partial}{\partial x}\left( f \circ (2y - x) \right)$$

rough $\left( h_1 \circ h_2 \, (x) \right)' =$

$h_1' \left( h_2(x) \right) \cdot h_2'(x)$

$h_1 = f$

$h_1' = \frac{\partial h_1}{\partial x} = \frac{\partial f}{\partial x} = f'$

$f : \mathbb{R} \to \mathbb{R}$ (one variable)

$$③ = f'(2y - x) \cdot \underbrace{\frac{\partial}{\partial x}(2y - x)}_{④}$$

$$④ = \underbrace{\frac{\partial}{\partial x}(2y)}_{=0} - \underbrace{\frac{\partial x}{\partial x}}_{1} = -1$$

As a result ; $\frac{\partial h}{\partial x}(x,y) =$

$$\sin(y)\left[\ln\left[f(2y-x)\right] - \frac{x}{f(2y-x)} \cdot f'(2y-x)\right] \cdot$$

$$e^{x\sin(y)\ln\left[f(2y-x)\right]}$$

$\frac{\partial h}{\partial x}\left(\pi, \frac{\pi}{2}\right) = -\pi$ taking into account that $f(0) = 1$ and $f'(0) = 1$.

student's answer

$\partial$

$\partial$

$$\boxed{\frac{d f(2y-x)}{d x}}$$

has not computed this

student's answer

$$\frac{\partial \boxed{f(2y-x)}}{\partial x} = f'(2y-x)$$

student's answer

$$\frac{\partial h}{\partial x}\left(\pi, \frac{\pi}{2}\right) = \frac{\partial}{\partial x}\left(\underbrace{\left[f\left(2\cdot\frac{\pi}{2}-\pi\right)\right]^{\pi \sin\left(\frac{\pi}{2}\right)}}_{\text{constant}}\right)$$

$$= 0$$